



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

THE RECONSTRUCTION OF THE MATHEMATICAL REQUIREMENT.

BY GEO. W. EVANS.

The following statement contains no material that has been decided upon by the Committee on the Curriculum in Algebra. I must assume personal responsibility for any mistakes of judgment.

In preparing this study of plans for the teaching of elementary mathematics, a careful survey has been made (1) of the report on foreign curricula made by the International Commission on the Teaching of Mathematics; (2) of the reports and discussions of the General Teaching Committee of the Mathematical Association of Great Britain; and (3) of reports and discussions in the United States. In general terms, this survey forces the conclusion that the constructive unrest so obvious in this country is not an isolated phenomenon, nor is a very radical reconstruction of our own plans likely to mark us as a peculiar people.

Except in Great Britain, the study of mathematics in the years corresponding to our seventh, eighth, and ninth grades has long been much more extensive, and at the same time much more concrete, than with us. Measurement and computation from models, the use of the pantograph in the study of similar figures, even the use of surveying instruments, is everywhere included in the program. Geometry and arithmetic are closely related, algebra and geometry are studied in the same years (in weekly or semi-weekly alternations, not "fused"). Applications of each subject are sought in the others. Approximate computation is included. Tables are used for interest and for similar matters. Logarithms are taught in the ninth grade or earlier. Continuous emphasis is laid on the function idea.

In Great Britain within fifteen years the actual text of Euclid has been disappearing from the schools. Examinations for the universities have been so liberally framed that it is now not only possible but indispensable to teach geometry as a science, not as an historic document. The most searching and stimulating book

in the world on the teaching of elementary algebra has been written by Dr. T. P. Nunn, and has been widely read, there and here. The subject of approximate computation as a school study has been created, and is now taught throughout England. Finally, in spite of the fact that English arithmetic is complicated by an antiquated and cumbrous monetary system, time is found to accomplish much more than our American schools at present can.

At the same time, and doubtless under the stress of war-time conditions, there is pressure for wider and more radical reform. A mathematician of world-wide authority* speaks of "the degeneration of algebra into gibberish," and goes on to say that the pupils "have got to be made to feel that they are studying something and not merely executing intellectual minuets."

Although what achievement there is belongs to the present century, the agitation in England goes back to mid-Victorian times, when the author of "Alice in Wonderland" wrote "Euclid and His Modern Rivals." There is a heartening echo from those early controversies in a letter from the Venerable J. M. Wilson, said to be the Nostradamus ridiculed by Dodgson. In this letter, dated in 1911, the veteran reformer reminds his successors that it is a syllabus for schools they are writing, and not a philosophic basis for algebra. He recommends in italics that the pupils be led to know mathematics as defined in the words of Comte, that is, as "the science of indirect measurement of magnitude, and the processes subsidiary thereto."

It must be confessed that some part of the reputed failure of mathematical teaching is to be attributed to the lack of unity commented on by the two personages last referred to. When we turn to the proposals for reform in the United States, taking as typical the Report on Algebra of the Missouri Society of Teachers of Mathematics and Science (1908), and the bulletin of the University of Texas on the Teaching of Geometry (1912), we find much excellent advice on the separate topics, lists of what we had better omit, suggestions as to approximate arithmetic, sources of problems, and so on, but no unifying general idea, no answer to the inevitable question, "What is it for?" To quote Whitehead again, "The pupils are bewildered by a

* A. N. Whitehead, in the *Mathematical Gazette*, January, 1916.

multiplicity of detail, without apparent relevance either to great ideas or to ordinary thoughts."

There is an interesting comment on the results of our American teaching from one of our French military visitors. He speaks of the student's *loyauté d'esprit*,—"no pose or touch of vanity, a disposition not to appear wiser than he really is lest he lose a chance of learning. *But*, there is an inordinate thirst for details, for separate definite facts, such as could be recorded in experience. This French officer had previously been a lecturer in literature at Johns Hopkins, and had found there the same tendency, the same "*illusion de savoir*," based on the accumulation of facts. He contrasts the education of the French student, who receives from the beginning an inheritance and tradition of general ideas. He is fed upon them, he loves them, at times he misuses them. In America, on the other hand, premature specialization predominates over general culture.

This difference appears clearly in a prolonged course of instruction. While with the French boy every bit of knowledge crystallizes about ideas already required, with the American new facts and ideas are fitted together like the stones in a mosaic, like the specimens in a museum. With the one, new knowledge is incorporated into a system already established, taking the place proper to its real importance; with the other, there is not always an effective criterion of structural place or relative importance.

He sums up his comment by the following statement, which might well be adopted as a pedagogical maxim in our secondary school mathematics:

"Surtout, n'enseignons des détails et des procédés pratiques qu'en les rattachant toujours aux quelques idées générales qui doivent être la base de notre enseignement."*

It is the "great ideas" that will give unity to our teaching; it is the "ordinary thoughts" that we seek in concrete applications. The untaught outsider considers mathematics remote and arid; if our pupils think so, wherewithal shall they be saved?

The United States is alone in hesitating to include numerical trigonometry in the syllabus of elementary mathematics. There

* André Morize, "Impressions d'un instructeur militaire français," *Harvard Graduates' Magazine*, March, 1918.

is every reason why we should hesitate if this is simply to add another detail to our apparently crowded and heterogeneous list. But it obviously satisfies the requirement of concreteness, of connection with "ordinary thought." It is also an extension of the "great idea" of similarity. Will it crowd out other details of equal or greater importance? What shall be our criterion in choosing the items to retain? In other words, what do we mean by "important"?

There is general agreement that the high school course in mathematics "should be planned mainly for the students who never go to a university or college."* At the same time, since many of the students who take this course will go to college, it is certainly desirable to plan it so that another course can succeed it, giving without duplication the additional topics and training, if any, that will be required for admission to college. We may refer to these two courses as the "general mathematics course," for students who may or may not be going to college, and the "special mathematics course," or courses, for pupils who need further study in preparation for college, or for technical pursuits such as engineering, or commerce and finance.

This accords in the main with the aims of reforms demanded everywhere. They are well stated by Professor E. W. Hobson, F.R.S., in his presidential address before the British Mathematical Association in January, 1912. Democratization of Education is the formula he uses; and by it he means, not the extension of education to wider classes of the population, but rather the adaptation of educational methods to the *intellectual democracy*; that is, such a transformation of method and matter as to meet the needs of those who are lacking in exceptional capacity in relation to this particular subject. He gives a statement of the organization of subject matter, and the ideals underlying it, as it stood in England until within a few years. Without quoting it, I venture to say that many of our critics would find it an excellent basis for their animadversions.

What great idea can be used in the general mathematics course as the main intent of study? How shall we connect these great ideas with the ordinary thoughts of the pupil's life? The answer to these two questions will enable us to decide on the importance of details.

* Missouri Report, 1908, p. 2.

"The purpose of the first-year course should be to train pupils in the solution of problems by means of the equation, rather than to exercise them in abstract manipulations."*

"One of the main aims of the course in algebra is to develop the idea of functionality, and the various items of the syllabus should be treated with this end in view."†

"Algebra, well taught, helps to acquaint the pupil with the process of generalization, and to beget a clear consciousness of conditions under which the process may or may not be valid. . . . It introduces the idea of quantity changing continuously, and of the functional dependence of one quantity upon another. . . ."‡

"The central topic of algebra is, beyond question, the equation and its applications."§

"Applied problems, or, as they are often called, reading problems, form possibly one of the most important topics of elementary algebra."||

The one great idea is that of the functional relations; but it is out of the question to present that idea to the pupil except as the fruit of a considerable mathematical experience. It is not that the idea is in itself difficult; on the contrary, it is so simple that the pupil is likely to see in it nothing fruitful. Like all abstractions that have come late to the mind of humanity, it cannot be stated as the basis for youthful study, but must come as a generalization when there is sufficient material for its induction, and when it need not remain inert for lack of concepts that it may serve to illuminate. All mathematical instructions must bear towards this as a goal. The teacher must have it in mind, and the pupil's advancement must be consciously directed along lines that will present first one and then another of the details upon which finally this great generalization can be based.

Another idea, a "great idea," to be sure, and not quite so remote as the function idea, is that of generalization. That, also, is to be in the mind of the teacher, but would surely be meaningless to the pupil until after a good deal of algebraic work. A formula is a generalized solution. Yes, but the pupil will

* Central Association Report, 1907, p. 3.

† British Report on Mathematics for Girls, 1916, p. 6.

‡ British Report on Algebra and Trigonometry, 1911, pp. 2 and 3.

§ "The Teaching of Mathematics," J. W. A. Young, 1907, p. 302.

|| Schultze, "Teaching of Mathematics," p. 330.

think it a rule for arithmetical application rather than a pattern solution. He will come to the generalization point of view, later; but he will have seen it himself, then. Sudden revelations dazzle him.

The principle of substitution is only a restricted aspect of generalization.

There remains the equation. So far as school algebra is concerned, it is a symbolic statement of rather complicated numerical facts, which can be systematically transformed into one or more direct statements of numerical value. On the first day of this algebra work, the pupil confronts a sequence of equations offered to him as abbreviations of the successive steps of a verbal argument. The argument in its unabbreviated form must be intelligible to him, otherwise, of course, the symbols will not be. That is to say, the algebra is unnecessary. The problem can be done "by arithmetic." What is it for then? Only to get accustomed to the symbolism, so that the more difficult problems can afterwards be "explained" symbolically without being unintelligible. It is a generalization of method.

By means of problems suitably chosen, the equation can be followed through successive complications until we have completed its theory so far as elementary algebra is concerned. We have only to find the problems, and that we shall refer to later. A number of processes of transformation will have to be studied, such as the "four operations" for algebraic expressions, and so on. These topics can remain subsidiary, and need not be enlarged into exhaustive treatises. That restriction will exclude a great deal of manipulative work, which all our critics, even the friendly ones, agree upon as the main obstacle to intelligent progress as distinguished from the acquisition of mechanical facility.

Incidentally the equation compels the teaching of other things, —subsidiary, but of fundamental importance. For one, negative numbers. You have to deal with them when they appear as values of the unknown letter. You can postpone the evil day by saying that the problem was stated in error, but the natural impulse to generalization, which algebra fosters, will eventually force the issue. With quadratics it is unavoidable. There is one false start which should always be avoided, but almost never

is. That is the equation whose solution is positive, but on account of a foolish rule of transposition presents a negative number on one side (the right side, if you please), and x with a negative coefficient on the other. That needless perplexity should always be avoided by adding enough to each side to get rid of all negative terms, and by putting confidence in the perfect symmetry of the equation sign. It isn't new, for Mohammed Ben Musa did it; so did Diophantus.*

Conceding that the equation serves very well as a unifying topic for algebra, let us consider what would be of corresponding worth in geometry. There is, in the first place, the notion of congruence, derived from our experiences with material things; if a machine fits together here, it will also fit together wherever we carry it,—even in an *aéroplane* miles above the earth, or in a mine, or in any land, however distant. What is the easiest way of deciding, by measurement, when material solids will fit together? Again, we see about us many structures which hold shape permanently. Can our geometry decide what parts in such a structure need to be devised to hold it together? Compare the jointed parallelogram and the jointed triangle. One is not permanent in form, the other is. How decide?

Again, there is the notion of similarity, upon which are based all maps and plans and models. In making a reduced copy of a plan, if all distances are drawn to scale, the angles in the new plan will all be the same as in the original. If we make all angles in the new plan the same as in the original, all the distances will be drawn to scale. If we construct a model of a tank, say to the scale of one eighth of an inch to a foot, the depth of the tank will be 96 times as great as the depth of the model, but the capacity of the tank will be 884,736 times as great as the capacity of the model.

The elementary course in geometry will be a scientific study of these two notions. The general aim will be to secure convenient and well-reasoned methods of indirect measurement.

Since this scientific study involves numerical statements more or less complicated, there will arise occasion for the use of algebraic equations, not only in the discussion of these methods of measurement, but also in problems arising from them.

* T. L. Heath, "Diophantus of Alexandria," 185, p. 89.

Accompanying all this work, in algebra as well as in geometry, should be a careful study of the accuracy attainable from the numerical data available. From this study the pupil not only obtains much needed practice in computation, but also constant occasion for self-reliant judgment and for a critical attitude towards his own results. Systematic methods of checking all computations should be carefully reasoned out for these approximate methods, and should be rigorously used in all cases. Checking our results should not be regarded as a minor matter to be resorted to only when our computation for the check is comparatively easy; on the contrary, if accuracy can only be assured by independent computation fully as difficult as the original work, it should be carried through without fail. Surveyors are often required to swear to their results in courts of law; navigators risk valuable property, and even human lives, on their accuracy of computation; tall buildings and bridges depend for their stability not only on the solidity of their foundations, but quite as much on the hundreds, and even thousands of separate computations by which the details of these structures are determined.

We have then these four topics which we may use in unifying the general course in mathematics: (1) Approximate computation, (2) the equation as a means of solving problems, (3) congruence, (4) similarity.

We shall find, I think, that most of the content of elementary algebra and of plane geometry would be included, except the very things that are most criticised on account of their formalism. Certain other details of a rather complicated character, such as the binomial theorem and geometric series, though undoubtedly useful and important, would have to be postponed as too technical for this introductory course. The complete schedulization of available topics is a matter of detail that will have to be left for a later day, and for more well-considered and widely discussed formulation.

CHARLESTOWN HIGH SCHOOL,
BOSTON, MASS.